Electric Ducted Fan Theory

This paper describes a simple theory of a ducted fan. It is assumed that the reader knows what it is an electric ducted fan (EDF), how it works, and what it is good for.

When I was badly bitten by the EDF bug and started searching the Internet for information to my great surprise I found that there is not much of it, at least not in the form that would satisfy my desire for a deeper understanding to EDF. There are papers on the large fans for man carrying aircraft and/or hovercraft but these do not touch the ducting. On the other hand, being a process engineer by trade I am aware that almost every heating, ventilation and air conditioning (HVAC) system contains a kind of EDF, including the ducting. So I decided to have my own look into the EDF.

The text below contains a lot of formulas and numbers which is, I fancy saying, a way we engineers see the world around us. It may deter a reader not accustomed to this way, sorry for that.

All the calculation use SI units, i.e. meter (m), second (s), kilogram (kg), and then newton (N) for force, pascal (Pa) for pressure, and watt (W) for power. Even if a variable is given in millimetres for convenience, it is entered into a formula in metres.

Further, a symbol π designates a ratio of circle circumference and its diameter (3.14159...), and a symbol g represents an acceleration of gravity (9.81 m/s2).

A fluid (air) is considered incompressible.

Any theoretical explanation is immediately followed by a sample calculation for a better clarity.

Written in August 2017 during my holiday in Bulgaria, Sunny Coast

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1. Basics of powered flight

When in equilibrium in horizontal flight then in vertical direction an aircraft weight is balanced by a lift force. In horizontal direction the aircraft drag is balanced by a thrust.

Flight is a dynamic process, and it depends on supplying energy. When the energy supply is lost an aircraft inevitably decelerate and/or descent.

An aerodynamic force can only be created by "taking" some air and accelerating it in a desired direction. For example, a wing takes a lot of air and pushes it down, thus creating the lift force. A bad news about this is that the air has been still before this wing arrived, and it is moving when the wing is gone, in other words, the aircraft left some energy behind that needs to supply afresh from the propulsion.

The same applies for the propulsion itself; if it is supposed to generate thrust it shall blow air backward at a velocity higher than is the speed of flight.

Following formulas apply:

$$T = m \cdot (v_e - v_{fs}) \qquad \text{(Eq. 1.1)}$$

$$P_{prop} = T \cdot v_{fs} \qquad \text{(Eq. 1.2)}$$

$$\eta_{prop} = \frac{1}{1 + \frac{v_e - v_{fs}}{2 \cdot v_{fs}}} \qquad \text{(Eq. 1.3)}$$

$$P_{drive} = \frac{P_{prop}}{\eta_{prop}} \qquad \text{(Eq. 1.4)}$$

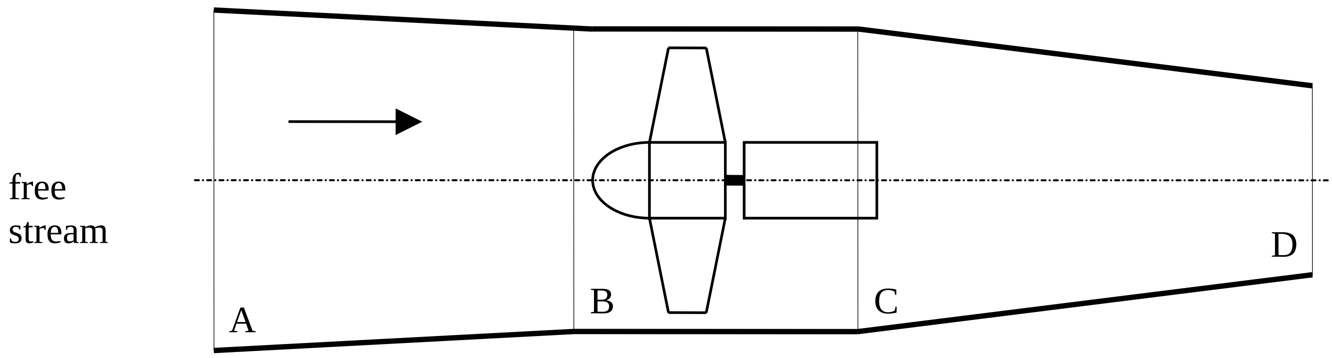
where

Symbol	*Unit*	*Description*
T	N	thrust
m	kg/s	mass flow-rate accelerates
ve	m/s	exhaust velocity
vfs	m/s	free stream velocity (speed of light)
Pprop	W	propulsion power

Symbol	*Unit*	*Description*
ηprop		propulsion efficiency, assuming that transfer of energy into air happens at middle between ve and vfs, while removal of energy at vfs
Pdrive	W	power supplied by drive system

2. Ducting

The electric ducted fan is an electrically driven propeller enclosed within a duct. The fan itself is in this chapter regarded as a black box that simply adds energy to the passing stream. The fan will be dealt with in the next chapter.



Five stations of interest are:
- free stream
- intake duct inlet (A)
- fan inlet (B)
- fan outlet (C)
- exhaust outlet (D)

Both intake duct and exhaust duct are shown as convergent; the cross-section at the fan is constant.

2.1 Geometry

An example fan (with a duct) has following parameters:

Parameter	*Symbol*	*Value*	*Unit*
Large diameter	D	65	mm
Small diameter	d	32	mm
Blade width	b	12	mm

Parameter	*Symbol*	*Value*	*Unit*
Number of blades	nl	7	
Blade pitch angle	eps	28	deg
Fan speed	N	36000	rpm
Intake cross-section ratio	Ai%	110	%FSA
Exist cross-section ratio	Ae%	90	%FSA
Intake duct length	Li	300	mm
Exhaust duct length	Le	150	mm

$$FSA = \frac{\pi}{4}\left(D^2 - d^2\right) \qquad \text{(Eq. 2.1)}$$

$$Ai = FSA \cdot Ai\% \qquad \text{(Eq. 2.2)}$$

$$Ae = FSA \cdot Ae\% \qquad \text{(Eq. 2.3)}$$

where

Symbol	*Unit*	*Description*
FSA	m2	Fan swept area, or a duct cross-section area at the fan location
Ai	m2	Intake cross-section area
Ae	m2	Exhaust cross-section area

$$FSA = \frac{\pi}{4}\left(0.065^2 - 0.032^2\right) = 0.00251 \text{ m}^2$$

$$A_i = 0.00251 \cdot 1.1 = 0.00277 \text{ m}^2$$

$$A_e = 0.00251 \cdot 0.9 = 0.00226 \text{ m}^2$$

2.2 Conservation of mass

There is no additional path for air for either entering or leaving the duct. This means that what enters at the intake station must leave at the exhaust station. As the fluid is considered incompressible the velocity at each station is indirectly proportional to its cross-section area.

$$m = A \cdot \rho \cdot v \qquad \text{(Eq. 2.4)}$$

where

Symbol	*Unit*	*Description*
A	m2	Station cross-section area
ρ	kg/m3	Air density = 1.2
v	m/s	Velocity
m	kg/s	mass flow-rate

The mass flow is chosen to be 0.138 kg/s at a plane forward velocity of 30 m/s for the calculated example. This choice will be explained further below.

Then the velocity at the fan is

$$V = \frac{m}{\rho \cdot FSA} = \frac{0.138}{1.2 \cdot 0.00251} = 45.8 \text{ m/s}$$

Intake velocity is then Vi = 41.6 m/s and exhaust velocity is Ve = 50.8 m/s.

The air stream velocities at particular stations are thus as follows:

Station	*Velocity*
free stream	30 m/s
intake duct inlet (A)	41.6 m/s
fan inlet (B)	45.8 m/s
fan outlet (C)	45.8 m/s
exhaust outlet (D)	50.8 m/s

2.3 Conservation of energy

2.3.1 Ideal (no loss) case

As with the mass also the stream energy is conserved in the intake duct and exhaust duct.

The energy of stream flowing within a duct can be expressed by a well-known Bernoulli theorem:

$$p + \frac{1}{2} \cdot \rho \cdot v^2 = const \qquad \text{(Eq. 2.5)}$$

Symbol	*Unit*	*Description*
p	Pa	static pressure
$0.5\rho v^2$	Pa	dynamic pressure

Thus, for two stations X and Y following stays true:

$$p_X + \frac{1}{2} \cdot \rho \cdot v_X^2 = p_Y + \frac{1}{2} \cdot \rho \cdot v_Y^2 \qquad \text{(Eq. 2.6)}$$

or

$$\Delta p = p_X - p_Y = \frac{1}{2} \cdot \rho \cdot \left(v_Y^2 - v_X^2\right) \qquad \text{(Eq. 2.7)}$$

Note: It is well known to modellers to trade off a speed for a height and vice versa. The Eq. 3 says exactly the same, only replaces "height" with "pressure".

As air is accelerated along the duct the pressure downstream the duct shall be lower than that at the inlet. But the static pressure is the same before and after the duct - that of the surrounding atmosphere. On the other hand, there is the fan in the duct that adds energy to the passing stream. It adds exactly the amount needed for reaching atmospheric pressure at the outlet.

Thus, the Eq. 2.7 could be re-indexed as

$$\Delta p_{fan} = \frac{1}{2} \cdot \rho \cdot \left(v_e^2 - v_{fs}^2\right) \qquad \text{(Eq. 2.8)}$$

$$\Delta p_{fan} = \frac{1}{2} \cdot 1.2 \cdot \left(50.8^2 - 30^2\right) = 1008 \,\text{Pa}$$

This would apply in an ideal world of no losses. In reality, the formula should like like

$$\Delta p_{fan} = \frac{1}{2} \cdot \rho \cdot \left(v_e^2 - v_{fs}^2\right) + \Delta p_{loss} \qquad \text{(Eq. 2.9)}$$

2.3.2 Real case with losses

Unfortunately, there is no exact, or scientific, method for determining the losses in the ducting. On the other hand, there exists engineering handbooks containing measured loss coefficients for many possible flow conditions and resistances (e.g. Crane - http://www.craneco.com/category/200/Purchase-Flow-of-Fluids.html).

In my opinion, even the applicability of the engineering flow handbooks to EDF should be considered with a great care, unless somebody volunteers to carry out a detailed measurements or CFD studies, but there is nothing better at hand now.

This lack of information is aggravated by a fact that the proper design of the ducting is of highest importance for the EDF performance, as will be apparent soon.

$$\Delta p_{loss} = K \cdot \frac{1}{2} \cdot \rho \cdot v_r^2 \qquad \text{(Eq. 2.10)}$$

Lost pressure is expressed as a multiple (K) of a reference dynamic pressure ($0.5\rho v^2$). The reference velocity is chosen here to be the fan velocity. When doing the detailed calculation the different velocities along the duct should be taken into an account and the pressure loss integrated. However, as explained above, the uncertainties in the determination of loss coefficients allow for this simplification.

The loss coefficient is usually expressed as extended Darcy-Weisbach formula

$$K = \lambda \cdot \frac{L}{D} + \sum k \qquad \text{(Eq. 2.11)}$$

where

Symbol	*Unit*	*Description*
K		loss coefficient
λ	-	friction factor
L	m	duct length
D	m	duct hydraulic diameter, diameter calculated from FSA is used for both intake and exhaust
Σk		sum of "shape" flow resistances

The loss has two components - one is due to friction, the other is due to shapes other than straight pipe.

Following data are input

Parameter	*Symbol*	*Value*	*Unit*
Friction factor	λ	0.03	-
Σk for intake	ksii	0.3	-
Σk for intake	ksie	0.0	-

A smooth and "regular" pipe duct of circular cross-section is assumed. The intake duct shape resistance corresponds to acceleration around the intake lip and in bends in the duct ("trousers").

Then the loss coefficients for the intake and exhaust duct can be evaluated as follows:

$$Ki = 0.03 \cdot \frac{300}{56.6} + 0.3 = 0.46$$

$$Ke = 0.03 \cdot \frac{150}{56.6} + 0 = 0.08$$

The ducting pressure drop is then (for both intake and exhaust):

$$\Delta p_{loss} = (0.46 + 0.08) \cdot \frac{1}{2} \cdot 1.2 \cdot 45.8^2 = 680 \,\mathrm{Pa}$$

Accordingly, the fan has now to provide 1688 Pa (about 170 mm of water column) rather than 1008 Pa in a case of no losses. In other words, even the simple ducting places almost 70% more load on the fan!

Also, the atmospheric pressure is about 100 000 Pa. A very small pressure changes within the duct (say few percent of atmospheric pressure) allow for neglecting the air compressibility.

Using the described procedure the conditions in all stations can be calculated as follows:

Station	*Velocity*	*Pressure*
free stream	30 m/s	0 Pa
intake duct inlet (A)	41.6 m/s	-498 Pa
fan inlet (B)	45.8 m/s	-1296 Pa
fan outlet (C)	45.8 m/s	392 Pa
exhaust outlet (D)	50.8 m/s	0 Pa

2.3.3 Fan power

The Eq. 2.5 has been said to be a law of energy conservation while it is in fact written as pressure conservation. It can be re-ordered and multiplied by mass flow rate to get the power, as follows:

$$m \cdot \left(\frac{p}{\rho} + \frac{1}{2} \cdot v^2 \right) = const \qquad \text{(Eq. 2.12)}$$

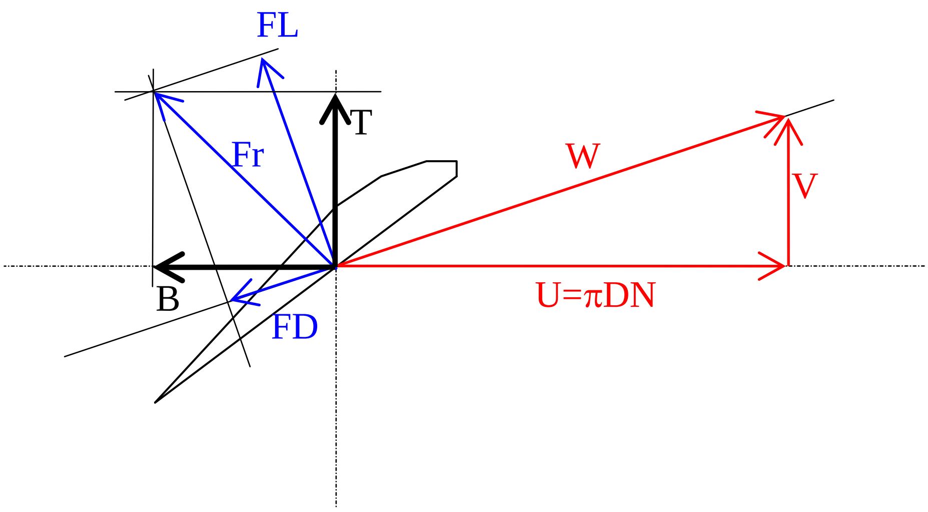
Considering that velocities at the fan inlet and outlet are the same and using the above logic the fan (still a black box) power could be calculated:

$$P_{fan} = m \cdot \left(\frac{\Delta p_{fan}}{\rho} \right) \qquad \text{(Eq. 2.13)}$$

$$P_{fan} = 0.138 \cdot \left(\frac{1688}{1.2} \right) = 194\,\mathrm{W}$$

3. Fan calculation

3.1 Forces acting on impeller blade



The blade is moving tangentially with a velocity U and axially with a velocity V. An aerodynamic force Fr is thus created on the blade. This force could be decomposed into a thrust T and braking force B. The blade input power is then a product of the tangential velocity and the braking force while the blade useful power is the product of the thrust and the axial velocity.

$$w = \sqrt{V^2 + (\pi \cdot d_r \cdot N)^2} \quad \text{(Eq. 3.1)}$$

$$\tan(\beta) = \frac{V}{\pi \cdot d_r \cdot N} \quad \text{(Eq. 3.2)}$$

$$F_L = c_L \cdot A_B \cdot \frac{1}{2} \cdot \rho \cdot w^2 \quad \text{(Eq. 3.3)}$$

$$F_D = c_D \cdot A_B \cdot \frac{1}{2} \cdot \rho \cdot w^2 \quad \text{(Eq. 3.4)}$$

$$T = F_L \cdot \cos(\beta) - F_D \cdot \sin(\beta) \quad \text{(Eq. 3.5)}$$

$$B = F_L \cdot \sin(\beta) + F_B \cdot \cos(\beta) \quad \text{(Eq. 3.6)}$$

$$P_{fan} = T \cdot V \quad \text{(Eq. 3.7)}$$

$$P_{shaft} = B \cdot \pi \cdot d_r \cdot N \quad \text{(Eq. 3.8)}$$

where

Symbol	*Unit*	*Description*
w	m/s	blade composite velocity
u	m/s	blade tangential velocity
V	m/s	blade axial velocity (air velocity at fan station)
N	1/s	fan speed
dr	m	blade reference diameter (middle point considered)
Ab	m2	blade(s) surface area
cl		blade airfoil coefficient of lift
cd		blade airfoil coefficient of drag
Pfan	W	fan power
Pshaft	W	fan input power

3.2 Airfoil polar

A NACA 4412 polar at Re = 100 000 has been calculated using the XFOIL code. The drag coefficient used in the calculation is larger than for the airfoil only by a factor of 3. This multiplier should account for drag components other than the profile drag.

3.3 Sample calculation

The large fan diameter is 65 mm, the small diameter is 32 mm. Considering 1 mm gap between the blade tip and the shroud, the blade length is 15.5 mm. The blade width is 12 mm, and there are 7 blades in the impeller. Accordingly, the blade surface Ab = 1302 mm2, and the reference diameter is 48.5 mm.

$$w = \sqrt{45.8^2 + (\pi \cdot 0.0485 \cdot 36000/60)^2} = 102.2 \text{ m/s}$$

$$\beta = \arctan\left(\frac{45.8}{\pi \cdot 0.0485 \cdot 36000/60}\right) = 26.6\,\text{deg}$$

The blade pitch angle is 28 degrees; accordingly the blade angle of attack is
28-26.6 = 1.4 deg. The airfoil NACA 4412 has at this AoA cl = 0.61 and cd = 0.053

Then

$$F_L = 0.61 \cdot 0.001302 \cdot 0.5 \cdot 1.2 \cdot 102.2^2 = 5.0 \text{ N}$$

$$F_D = 0.159 \cdot 0.001302 \cdot 0.5 \cdot 1.2 \cdot 102.2^2 = 0.43 \text{ N}$$

$$T = 5.0 \cdot \cos(26.6) - 0.43 \cdot \sin(26.6) = 4.3\,\text{N}$$

$$B = 5.0 \cdot \sin(26.6) + 0.43 \cdot \cos(26.6) = 2.6\,\text{N}$$

$$P_{fan} = 4.3 \cdot 45.8 = 197\,\text{W}$$

$$P_{shaft} = 2.6 \cdot \pi \cdot 0.0485 \cdot 36000 / 60 = 238 \text{ W}$$

Please note that the fan power calculated from the propulsion considerations is the (almost) same as that calculated from the fan blades itself. In fact, the calculation program searches for a fan velocity V to match the power required by the ducting and that provided by the fan.

4. Efficiencies

Let's now consider the energy losses within the system.

The drive is "sucking" power Pbatt from the power source a converting it into the fan shaft power Pshaft with an efficiency of eta_motor. This efficiency covers all losses in the electrical components of the system, i.e. cables, connectors, speed regulator, and electric motor, and it is set to 80%. As calculated above, the fan consumes 238 W, accordingly the power drawn from the battery is 238/0.8 = 298 W.

The fan then accepts the Pshaft and converts it into Pfan with an efficiency of eta_fan. These losses are due to aerodynamics of the fan, the blades creating both lift and drag. As can be seen above, the fan efficiency is 197/238 = 83%

The air leaving the fan carries the energy Pfan but some of it is spent in overcoming the duct resistances. In our example the duct efficiency is ratio of the fan pressure differential for the "no-loss" case and "real" case, or 1008/1688 = 60%. The drive delivered power is thus 197*0.6 = 118 W.

Alternatively, this Pedf can be calculated from formula

$$P_{edf} = \frac{m}{2} \cdot \left(v_e^2 - v_{fs}^2\right) \qquad \text{(Eq. 4.1)}$$

$$P_{edf} = \frac{0.138}{2} \cdot \left(50.8^2 - 30^2\right) = 116 \text{ W}$$

But this is not all. The air is moving faster than the plane. Air behind the plane is thus stirred and this also means an energy loss (please refer to Chapter 1).

The EDF thrust can be calculated from (formulas already mentioned above)

$$T = m \cdot \left(v_e - v_{fs}\right) \quad \text{(Eq. 4.2)}$$

$$T = 0.138 \cdot (50.8 - 30) = 2.87 \text{ N}$$

The propulsion power is then

$$P_{prop} = T \cdot v_{fs} \quad \text{(Eq. 4.3)}$$

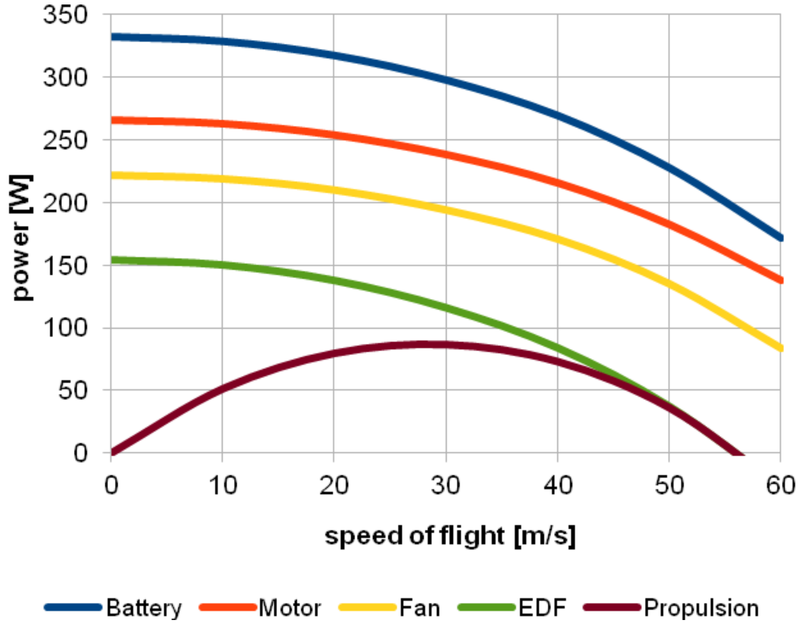
$$P_{prop} = 2.87 \cdot 30 = 86 \text{ W}$$

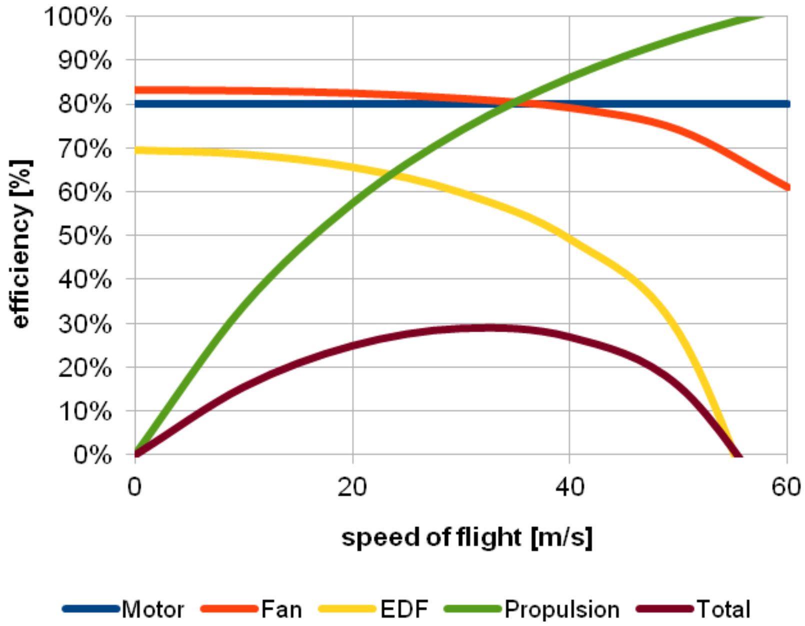
The propulsion efficiency is then 86/118 = 73%.

From 297 W input at the beginning we get 86 W at the end, the overall efficiency is thus 86/297 = 29%.

	Output power in [W]	*Efficiency*
battery	297	
motor	238	80%
fan	197	83%
EDF	118	60%
propulsion	86	73%

The following diagrams show the corresponding powers and efficiencies over the speed range for the discussed example.





5. Model airplane

Eventually, we should check the match between the EDF and the model.

The model can be characterised (at least) by the following parameters:

Parameter	*Symbol*	*Value*	*Unit*
Wing span	L	800	mm
Average wing chord	b	170	mm
Weight	M	800	g
Glide ratio at CL=1	ε	10	

It is assumed that the model will have the best glide ratio at CL (coefficient of lift) equal to 1. It is further assumed that the drag of the model consists of an induced drag and other drag. The induced drag changes with the lift as

$$c_{Di} = \frac{c_L^2}{\pi \cdot ar} \qquad \text{(Eq. 5.1)}$$

where

Symbol	*Unit*	*Description*
cdi		induced drag coefficient
cl		lift coefficient
ar		wing aspect ratio

The "other" drag is then independent of speed (assumed as such) and can be calculated as

$$c_{Do} = \frac{1}{\varepsilon} - \frac{1}{\pi \cdot ar} \qquad \text{(Eq. 5.2)}$$

Thus, for a given speed the lift coefficient is calculated as

$$c_L = \frac{2 \cdot M \cdot g}{\rho \cdot L \cdot b \cdot v^2} \qquad \text{(Eq. 5.3)}$$

Drag coefficient is calculated as

$$c_D = c_{Do} + \frac{c_L^2}{\pi \cdot ar} \qquad \text{(Eq. 5.4)}$$

And eventually drag force is evaluated as

$$F_D = \frac{M \cdot g \cdot c_D}{c_L} \qquad \text{(Eq. 5.5)}$$

Using the above data, the model wing aspect ratio is 800/170 = 4.7, and cdo = 0.032. Then for the speed v = 30 m/s

$$c_L = \frac{2 \cdot 0.8 \cdot 9.81}{1.2 \cdot 0.8 \cdot 0.17 \cdot 30^2} = 0.11$$

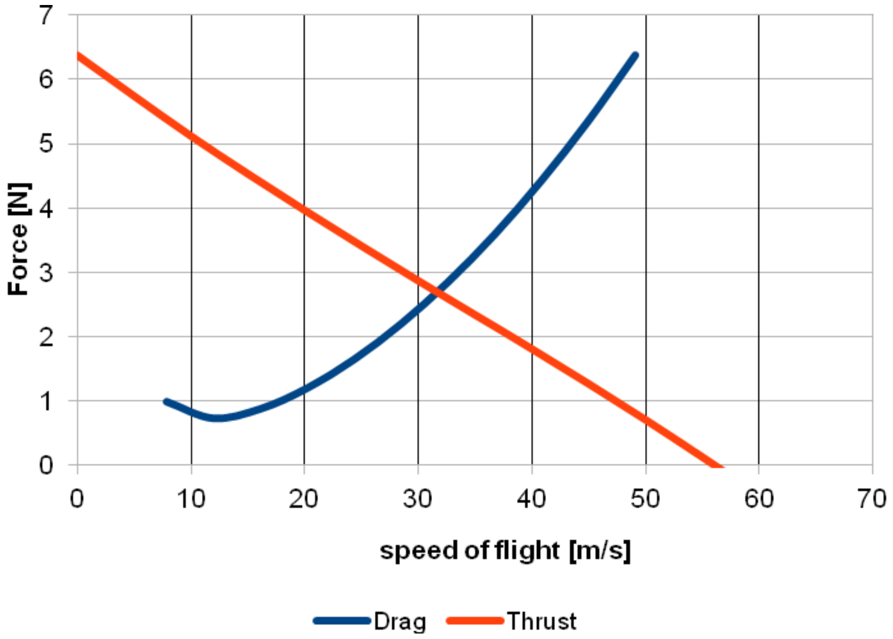
$$c_D = 0.032 + \frac{0.11^2}{\pi \cdot 4.7} = 0.033$$

Glide ratio at this speed is thus 0.11/0.033 = 3.33, and required thrust allowing flying at this speed is

$$F_D = \frac{0.8 \cdot 9.81 \cdot 0.033}{0.11} = 2.4 \text{ N}$$

The available thrust calculated from the Eq. 4.2 above is 2.9 N so the model should reach 30 m/s easily.

The model drag and EDF thrust as a function of the speed of flight is shown on the following diagram.



6. Summary and conclusions

A theory describing the electric ducted fan is presented. An example calculation is included, demonstrating the method.

Several assumptions have been made during the course about aerodynamic properties of the fan, ducting and airframe. These properties can be ascertained only by a laborious and detailed measurement of a particular model airplane and therefore the use of the described method is rather limited.

However, some conclusion could be made:

1) A buried propelled is a very inefficient way of propelling a model airplane. It does not make much sense (at least to me) to build a model if either speed or efficiency is of a primary concern. On the other hand, EDF means a jet (semi-) scale model and there is no other way of simulating a jet aircraft within a budget.

2) The ducting seems to be a critical point of each EDF design. The largest energy loss occurs in the duct, even though its design considered here was quite simple and "minimalistic". The ducting design and construction should be looked into in next steps, and it is my plan to do so in continuation to this paper.

That is all for now. Any comment/question/suggestion will be appreciated. Thank you, Jan